



Study on the Probability Distribution of Environmental Pollution Risk Caused by Sewage Irrigation in Farmland

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ABSTRACT

The shortage of water resources is the bottleneck that restricts China's economic and social development, especially the shortage of water in agriculture is even more serious. The intensification of industrial and domestic sewage discharges has led to the pollution of the water quality in all major water areas, and as a result, agriculture faces the serious challenge of water quality-induced water shortage. Within a long period of time, irrigation of farmlands with sewage is inevitable and there will be increase but no decrease. Though a lot of research has been done on the risk of environmental pollution caused by sewage irrigation, these studies do not have a deep and quantitative analysis of the degree of environmental risk caused by irrigation of farmlands with sewage. Based on the assumption that the degree of environmental risk caused by irrigation of farmlands with sewage can be correctly reflected, this paper studies the probability distribution of environmental risk caused by irrigation of farmlands with sewage through inference, calculus and non-parametric test, and by making use of the actual data, draws the conclusion through calculation that the probability distribution of the degree of environmental risk caused by irrigation of farmlands with sewage has normal distribution (0.275, 0.015).

INTRODUCTION

Irrigation of farmlands with sewage has become a common phenomenon in China's agricultural development (Maassen 2016). Sewage irrigation can provide water and fertilizers to the crops, but due to the low sewage treatment capacity, high sewage treatment cost and people's lack of awareness of environmental protection, a large amount of untreated sewage or sewage without necessary pre-treatment is used to irrigate farmlands, causing water quality to severely exceed the limit (Zeng et al. 2015, Jing et al. 2017). Blind sewage irrigation has resulted in the accumulation of toxic and harmful substances in soil and water sources in some areas to pollute the environment to various degrees (Liu et al. 2016, Liu et al. 2015, Wu 2018). The incidence of diseases such as cancer in many sewage irrigation areas is much higher than that in non-sewage irrigation areas, and such a trend is still on the rise (Meena et al. 2016). In recent years, people have attached more attention to the environmental pollution risk caused by sewage irrigation, and carried out a great deal of research as well and made a desirable amount of achievements (Xu et al. 2017, Lv et al. 2015, Verma et al. 2017). However, the research methods of sewage irrigation risks fail to conduct a penetrating and quantitative analysis of the degree of environmental risk caused by irrigation of farmlands with sewage, most

studies are unilateral longitudinal studies in their own fields and fewer studies have been done on the degree of various environmental risks caused by irrigation of farmlands with sewage, let alone studying and exploring the probability distribution of environment damage caused by sewage irrigation.

Without considering the conditions of non-sewage irrigation, this paper studies and discusses the probability distribution of environmental risk caused by irrigation of farmlands with sewage. Due to the randomness of annual precipitation, the degree of drought in agriculture is random every year. As a result, the amount of sewage needed to irrigate farmlands is random, and the risk arising from there is random as well. By using the agricultural drought model, this study can conclude from the probability distribution of annual precipitation the probability distribution of environmental risks caused by irrigation of farmlands with sewage as well as its digital characteristics.

PROBABILITY FUNCTION OF FARMLAND SEWAGE IRRIGATION RISKS

The annual precipitation distribution and population parameters as determined by the curve fitting method: Under the normal circumstances, annual precipitation obeys P-III distribution.

Pearson P-III curve: Assume that the random variable X obeys P-III distribution and its probability density function is:

$$f(x; \alpha, \beta, a_0) = \frac{\beta^\alpha}{\Gamma(\alpha)} \cdot (x - a_0)^{\alpha-1} \cdot e^{-\beta(x-a_0)} \quad \dots(1)$$

In the formula, $\Gamma(\alpha)$ is the gamma function of α ; α, β, α_0 are three parameters.

Obviously, after the three parameters are determined, the density function is determined accordingly. It can be inferred that these three parameters have the following relations with the three characteristic population parameters \bar{x}, C_v, C_s :

$$\alpha = \frac{4}{C_s^2}, \quad \beta = \frac{2}{\bar{x} C_v C_s}, \quad a_0 = \bar{x} \left(1 - \frac{2C_v}{C_s} \right) \quad \dots(2)$$

According to the physical properties of hydrological variables, it is easy to know that $\alpha > 0, \beta > 0, \alpha_0 \geq 0$. As long as the three parameters of \bar{x}, C_v , and C_s are determined, the probability density function of P-III distribution can be determined as well.

Parameter estimation in the curve-fitting method: The method in which parameters are selected optimally is based on the estimated frequency distribution curve and dot pitch distribution of sample experience is called the curve-fitting method (Ren et al. 2001). In this paper, the curve is determined by using this method, and the parameter of the curve is considered as the estimation of the population parameter, and then the design value of the hydrologic variable of the specified frequency is obtained.

Evaluate the pros and cons of parameter estimation: Most hydrological variables are infinite, and at present, the grasped observation data are only the sample with a small capacity. There always exists a certain deviation and error by estimating the overall characteristics and parameters with such samples. In respect of the parameters estimated by the curve-fitting method, the parameter values of fitting lines are different from person to person through ocular estimation, and the bias is irregular. However, the long-term accumulated practical experience shows that the mean value is generally unbiased, and the coefficient of variation and the coefficient of skew are generally positively biased.

Mathematical model of crop water production functions in the whole growth period: The mathematical model of crop water production functions in the whole growth period is generally a parabola form:

$$Y = a_0 + b_0 \cdot W + c_0 \cdot W^2 \quad \dots(3)$$

In the formula, Y is crop output; W is irrigation amount;

α_0, β_0, c_0 are empirical coefficients, according to the actual problem $c_0 < 0$.

Considering that the output is zero when the water supply is zero, the formula (3) is changed as:

$$Y = b_0 \cdot W_C + c_0 \cdot W_C^2 \quad \dots(4)$$

In the formula, W_C is the actual water supply of the crop; the meanings of the rest symbols are the same as above.

According to the principle of marginal benefit analysis, assume that Y_{\max} is the maximum yield of the crop under full irrigation; at this time, the corresponding water supply is W_N . When

$$W_C \leq W_N, \quad \frac{dY}{dW_C} > 0, \quad \frac{d^2Y}{dW_C^2} = 2c_0 < 0, \text{ namely } c_0 < 0. \text{ At this time:}$$

$$D_r = \frac{Y_{\max} - Y}{Y_{\max}} = \frac{(b_0 \cdot W_N + c_0 \cdot W_N^2) - (b_0 \cdot W_C + c_0 \cdot W_C^2)}{b_0 \cdot W_N + c_0 \cdot W_N^2} \\ = \frac{b_0 \cdot (W_N - W_C) + c_0 \cdot (W_N - W_C)(W_N + W_C)}{b_0 \cdot W_N + c_0 \cdot W_N^2}$$

Make $\frac{W_N - W_C}{W_N} = R_{SWS}, 0 \leq R_{SWS} \leq 1$ is called water stress of relative soil. When $W_C = W_N - W_N \cdot R_{SWS}$ is substituted into the above formula, then:

$$D_r = \frac{-c_0 \cdot W_N^2 \cdot R_{SWS} + W_N(2 \cdot W_N \cdot c_0 + b_0) \cdot R_{SWS}}{b_0 \cdot W_N + c_0 \cdot W_N^2} \\ = -\frac{c_0 \cdot W_N^2}{b_0 \cdot W_N + c_0 \cdot W_N^2} \cdot R_{SWS}^2 + \frac{W_N(2 \cdot W_N \cdot c_0 + b_0)}{b_0 \cdot W_N + c_0 \cdot W_N^2} R_{SWS} \\ = A \cdot R_{SWS}^2 + B \cdot R_{SWS} \quad \dots(5)$$

In the formula, $D_r(R_{SWS})$ is the output loss function of water stress in the whole growth period of crops.

$$A = -\frac{c_0 \cdot W_N^2}{b_0 \cdot W_N + c_0 \cdot W_N^2} > 0, B =$$

$$\frac{W_N(2 \cdot W_N \cdot c_0 + b_0)}{b_0 \cdot W_N + c_0 \cdot W_N^2}, A + B = 1$$

When, $R_{SWS} = 0, D_r = 0$; when $R_{SWS} = 1, D_r = A + B = 1$.

When the least square method is used to solve b_0 and c_0 , then:

$$b_0 = \frac{\sum_{i=1}^n Y_i W_i \cdot \sum_{i=1}^n W_i^4 - \sum_{i=1}^n Y_i W_i^2 \cdot \sum_{i=1}^n W_i^3}{\sum_{i=1}^n W_i^4 \cdot \sum_{i=1}^n W_i^2 - \left(\sum_{i=1}^n W_i^3 \right)^2} \\ c_0 = \frac{\sum_{i=1}^n Y_i W_i \cdot \sum_{i=1}^n W_i^3 - \sum_{i=1}^n Y_i W_i^2 \cdot \sum_{i=1}^n W_i^2}{\left(\sum_{i=1}^n W_i^3 \right)^2 - \sum_{i=1}^n W_i^4 \cdot \sum_{i=1}^n W_i^2} \quad \dots(6)$$

Available water supply:

$$W_C = P_0 + G + w_0 = \alpha \cdot R + G + w_0 \quad \dots(7)$$

In the formula, W_C is the available water supply of crops, mm; P_0 is the effective rainfall of crops, mm; α is the rainfall infiltration coefficient. Generally, it is considered that when rainfall capacity at one time is smaller than 5 mm, α is 0; when rainfall capacity at one time is between 5 ~ 50 mm, α is about 1.0 ~ 0.8; when rainfall capacity at one time is larger than 50 mm, α is about 0.7~0.8; R is the annual precipitation, mm; G is the underground water capacity of crops that can be resupplied, mm; w_0 is the initial water storage of crop soil, mm, which can be calculated by the following formula:

$$w_0 = 1000 \times H \times n \times \theta \quad \dots(8)$$

In the formula, H is the wet layer depth of the crop soil plan, m; n is the voidage of the soil in the planned wet layer (calculated by its % in the soil volume); θ is the moisture content of the crop soil (calculated by its % of the soil void volume).

Water requirement:

$$W_N = ET + w' \quad \dots(9)$$

In the formula, W_N is the water demand of crops, mm; ET is the evapotranspiration amount of crops, mm; w' is the allowed minimum soil water storage in the whole growth period of crops, and the desirable field water retention is 60% ~ 80%, mm.

Probability distribution: It can be deduced from Formula (3) to Formula (6) that:

$$D(R) = \begin{cases} A \cdot (1 - \frac{W_C}{W_N})^2 + B \cdot (1 - \frac{W_C}{W_N}), & 0 \leq W_C \leq W_N \\ 0, & W_C > W_N \end{cases} \quad \dots(10)$$

In the formula, $D(R)$ is the risk caused by irrigation of farmlands with sewage. Through the joint solution of Formula (7) and Formula (10), it can be known that it is the function of the random variable R ; the meanings of the rest symbols are the same as above.

Assume that the distribution density function of R is $\phi(r)$, then $F(x)$, namely the probability distribution function of D is $F(x)=P(D \geq x)$. It can be easily known that when $x > 1$, $F(x)=0$; when $x < 0$, $F(x)=1$; when $0 \leq x \leq 1$, from $D \geq x$ and by means of Formula (10), it can be drawn that $R \leq S(x)$:

$$F(x) = P(R \leq S(x)) = \int_0^{s(x)} \phi(r) dr$$

$f(x)$, namely the distribution density function of D is:

$$f(x) = \frac{dF(x)}{dx} = \phi(S(x)) \cdot \frac{dS}{dx}, 0 \leq x \leq 1; 0, \text{ others} \quad \dots(11)$$

$E(D)$, namely the expected value of D is:

$$E(D) = \int_0^1 x \cdot f(x) dx \quad \dots(12)$$

$V(D)$, namely the variance of D is:

$$V(D) = \int_0^1 [x - E(D)]^2 \cdot f(x) dx \quad \dots(13)$$

TESTING METHOD OF NON-PARAMETRIC ASSUMPTION

Using the real rainfall series data of the research area to calculate the risk series from Formula (3) to Formula (10), the Shapiro-Wilk W test can be performed to carry out a non-parametric assumption test on the risk samples. W test is a normal test, which requires the sample size n between 3~50. This test method can be adopted to check whether a batch of observation values or random numbers is from the same normal distribution. This test has been determined as a National Standard (Wu et al. 1995). The testing steps of W are as follows.

Step one: Arrange the observed values (random numbers) in a non descending order, namely $X_{(1)} \leq \dots \leq X_{(n)}$.

Step two: By formula

$$W = \frac{\left\{ \sum_{k=1}^{\left[\frac{n}{2} \right]} a_k(W) [X_{(n+1-k)} - X_{(k)}] \right\}^2}{\sum_{k=1}^n (X_{(k)} - \bar{X})^2} \quad \dots(14)$$

Calculate the value of the statistic W . In the formula above, $a_k(W)$ can be obtained by look-up table.

Step three: For the given significance level α and sample size n , W_α can be obtained by look-up table.

Step four: Make a judgment. If $W < W_\alpha$, then reject H_0 , otherwise do not reject H_0 .

EXAMPLE APPLICATION

The probability distribution of sewage irrigation was studied with the data of a sewage irrigation area, where the main crop in the irrigation area is paddy rice. According to the balance analysis of water resources, when the irrigation guarantee rate P is equal to 90%, the local rainfall runoff is difficult to meet the irrigation water requirements and thus irrigation is needed. From 10 years of meteorological, geological and hydrological data in the local place as well as the corresponding data of paddy rice in the whole growth period (May to September), the calculated and sorted data are given in Table 1.

The initial water storage of soil is 86.4 mm; and the water requirement of paddy rice in the whole growth period

Table 1: Parameters of P-III curve distribution of rainfall for paddy rice in the whole growth period.

Parameters	C_v	C_s	\bar{X}
Values	0.15	0.30	1359

is set to be $W_N=600$ mm. According to the two years known irrigation amount and the corresponding output of paddy rice, $b_0 = 1.5658$, $c_0 = -0.00065$; then $A = 0.329$, $B = 0.671$.

With the rainfall series within the whole growth period for 10 years in the local place, the data of 10 environmental risk degrees of sewage irrigation can be calculated, as given in Table 2.

The W test on the degree of environmental risk of sewage irrigation was conducted in order to facilitate the calculation (Table 3).

After calculating $W = 0.979$, at the significance level of $\alpha=0.01$, $\alpha=0.05$, $\alpha=0.10$ respectively, $W_{0.01} = 0.781$, $W_{0.05} = 0.842$, $W_{0.10} = 0.869$ were obtained by table look-up, and it was found that $W > W_{0.01}$, $W > W_{0.05}$, $W > W_{0.10}$. So H_0 is accepted, and it is considered that the environmental risk degrees of sewage irrigation in the local place can be regarded as obeying normal distribution under three kinds of significant level. The maximum likelihood estimation method was used to estimate the parameters, and environmental risk caused by irrigation of farmlands with sewage was found to be $\sim N(0.275, 0.015)$.

CONCLUSION

With the rapid development of China’s national economy and the enhancement of people’s living standards, agricultural irrigation water is constantly being squeezed by industrial and urban domestic water consumption, resulting in a growing shortage of water in agriculture. In the meanwhile, the intensification of industrial and domestic

sewage discharge has led to the pollution of water in all major areas of the country, and the agriculture has been seriously challenged by water quality-induced water shortage. Therefore, farmers have no choice but to apply a lot of sewage to irrigate farmlands. It can be expected that for a long period of time in the future, there is increase, but no decrease in the area of sewage irrigation. There has been a large number of studies on the hazard risk caused by sewage irrigation, however, most of these studies are quantitative studies on safety allowable values, but there are few studies on the probability distribution of environmental risks.

In view of the above problems, this paper employs the non-parametric test method to study and explore the probabilistic distribution of environmental risk caused by irrigation of farmlands with sewage, and finally by calculating the actual data of a sewage irrigation area, it concludes the probability distribution of environment risks caused by irrigation of farmlands with sewage in the local place.

The W test used in this paper can only test whether the data obey normal distribution. When data cannot be regarded as obeying the normal distribution, it is unable to determine which distribution it obeys. In addition, when data can be regarded as obeying the normal distribution, it may also be considered to obey another common distribution, such as the logarithmic distribution. At this time, it is necessary to make a comparison and judge the type of distribution more suitable for obedience, and this question remains to be further studied in the future.

REFERENCES

Jing, X., Yao, G.J., Liu, D.H., Liang, Y.R., Luo, M. and Zhou, Z.Q. 2017. Effects of wastewater irrigation and sewage sludge application on soil residues of chiral fungicide benalaxyl. *Environmental Pollution*, 224: 1-6.
 Lv, Y., Guan, X.Y., Ruan, B.Q. and Wang, Y.W. 2015. Multifractal

Table 2: Environmental risk degrees of sewage irrigation.

1	2	3	4	5	6	7	8	9	10
0.058	0.153	0.216	0.237	0.251	0.281	0.298	0.337	0.390	0.531

Table 3: Calculation list of the W test.

k	$X_{(k)}$	$X_{(11-k)}$	$X_{(11-k)}-X_{(k)}$	$a_k(W)$	$a_k(W)[X_{(11-k)}- X_{(k)}]$
1	0.058	0.531	0.473	0.5739	0.271455
2	0.153	0.390	0.237	0.3291	0.077997
3	0.216	0.337	0.121	0.2141	0.025906
4	0.237	0.298	0.061	0.1224	0.007466
5	0.251	0.281	0.03	0.0399	0.001197

- characteristics of soil particle size distribution under sewage irrigation in different irrigation years. *Applied Mechanics and Materials*, 3693(700): 205-210.
- Liu, Y., Wang, H.F., Li, X.T. and Chang, L.J. 2015. Heavy metal contamination of agricultural soils in Taiyuan, China. *Pedosphere*, 25(6): 901-909.
- Liu, B.L., Ma, X.W. and Ai, S.W. 2016. Spatial distribution and source identification of heavy metals in soils under different land uses in a sewage irrigation region, northwest China. *Journal of Soils and Sediments*, 16(5): 1547-1556.
- Maassen, S. 2016. Bibliometric analysis of research on waste water irrigation during 1991-2014. *Irrigation and Drainage*, 65(5): 644-653.
- Meena, R., Datta, S.P., Golui, D., Dwivedi, B.S. and Meena, M.C. 2016. Long-term impact of sewage irrigation on soil properties and assessing risk in relation to transfer of metals to human food chain. *Environmental Science and Pollution Research*, 23(14): 14269-14283.
- Ren, S.M., Zhu, Z.Y. and Zhang, W.P. 2001. *Engineering Hydrology*. China Agricultural University Press, Beijing.
- Verma, K., Gupta, A.B. and Singh, A. 2017. Optimization of chlorination process and analysis of THMs to mitigate ill effects of sewage irrigation. *Journal of Environmental Chemical Engineering*, 5(4): 3540-3549.
- Wu, Y., Li, Y.L. and Hu, Q.J. 1995. *Applied Mathematical Statistics*. National University of Defense Technology Press, Changsha.
- Wu, B., Guo, S.H., Li, X.J. and Wang, J. 2018. Temporal and spatial variations of polycyclic aromatic hydrocarbons (PAHs) in soils from a typical organic sewage irrigation area. *Science of the Total Environment*, 613-614: 513-520.
- Xu, S.S., Hou, P.F. and Xue, L.H. 2017. Treated domestic sewage irrigation significantly decreased the CH₄, N₂O and NH₃ emissions from paddy fields with straw incorporation. *Atmospheric Environment*, 169: 1-10.
- Zeng, X.F., Wang, Z.W., Wang, J., Guo, J.T., Chen, X.J. and Zhuang, J. 2015. Health risk assessment of heavy metals via dietary intake of wheat grown in Tianjin sewage irrigation area. *Ecotoxicology*, 24(10): 2115-2124.